Digital Dice: Computational Solutions to Practical Probability Problems,
by Paul J. Nahin,
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Wow! Few authors can hope to match the style of this book: clear, entertaining, and witty. The reader is immediately motivated to dig deeper.

Physicist Richard Feynman famously observed "What I cannot create, I do not understand." In that spirit, Nahin proposes that the reader experiment with a series of twenty-one beautifully chosen problems in probability. All the problems arise from everyday life; and most can be 'solved' with a Monte-Carlo simulation.

Nahin has three audiences in mind: "teachers of either probability or computer science looking for supplementary material for use in their classes, students in those classes looking for additional study examples, and aficionados of recreational mathematics...." In a stimulating introduction, Nahin lays out the book's philosophy; lets you check, using an amusing little anecdote, that you satisfy the 'prerequisites' needed to understand this book; and then explains, using two geometric probability problems, what computer simulations are.

Now, on to the fun stuff: the twenty-one problems. Nahin chose them not only for their pedagogical content, but also because of the engaging stories associated with many of them. As Nahin explains, Problem 4, *A curious coin-flipping game*, defied solution for a quarter-century. Problem 8, *A Toilet Paper Dilemma*, has achieved minor cult status as few Toilet Paper Problems ever have. The results of Problem 19, *Electing Emperors and Popes*, suggest that two reported events on historical elections of Popes are most unlikely actually to have occurred. Problem 16, *The Appeals Court Paradox*, lets you explore the probability that a Court errs, and how this probability changes if the worst judge decides to always follow the lead of the best judge.

Some problems can serve as an introduction to active fields of research, such as queueing theory in Problem 15, *How Long Is the Wait to Get the Potato Salad?* Here the reader is asked to simulate the operation of a deli counter: customers present themselves randomly at the counter, at an easily measured average rate (say λ customers per hour), and the deli clerk takes various amount of time to fill the various orders. The service time for each customer is again a random quantity, but with an easily measured average rate of service (say μ customers per hour). The reader is asked to help the store management in figuring out answers to such mathematical questions as what is the average total time at the deli counter for the customer (total time is the sum of the waiting time and the service time), and what is the maximum total time experienced by the unluckiest of the customers. What happens to these questions if a second, equally skilled deli clerk is hired? For this, the reader will need to write a computer simulation of a 10-hour day at the deli. This certainly involves getting down to the nitty-gritty of algorithm development, an important issue emphasized throughout the book. In contrast to most problems in the book, here a crucial question needs to be answered before the simulation can be done. What should one use in this problem to simulate the random time between the arrivals of Customer i and Customer $i+1$? Nahin suggests $-\log(\text{rand})/\lambda$, where *rand* is the uniform random variable; and he invites his readers to look up Poisson queues (not to be confused with the French 'queues de poisson') in any good book on stochastic processes or operations research to learn the theorerical underpinnings of this suggestion.

Solutions to each problem include the complete computer code in MATLAB, and useful references to the literature. Several theoretical discussions are given in the appendices, such as Appendix 2 on evaluating the results of a Monte Carlo simulation. A glossary of terminology is also included, making this book very user-friendly.

The reader will find in *Digital Dice* many examples of how much of mathematics really is done: somebody gets an interesting idea and does some experimentation (here a computer simulation), which is later followed by a theoretical confirmation (proof). Such examples are especially important to beginners in the subject. It is often very difficult to teach students how to experiment: more books such as this one, in other fields of mathematics, are waiting to be written. In the early nineteenth century, C.F. Gauss was able to gain deep insight into problems through his exceptional powers of computation. Nowadays, the power of computation is available to mathematicians who master the power of computers. No mathematics majors should graduate without a working knowledge of computer simulations. This delightful book provides ample incentive to gain that knowledge.

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