Intersection Matrices of Wild Quotient Singularities

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If the ring B is not regular at the maximal ideal $\mathfrak{m}_B$, the closed point $\mathfrak{m}_B$ of $\mathrm{Spec}(B)$ is called a cyclic quotient singularity. The singularity is called wild when $p = \mathrm{char}(k)$.

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Problem (c). In the presence of a quotient singularity on $\mathrm{Spec}(B)$, describe a desingularization $\pi : X \to \mathrm{Spec}(B)$ of the closed point $\mathfrak{m}_B$.

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Main problem in this talk. For each prime p, describe the set of all possible intersection matrices N that can arise from the desingularization of a wild $\mathbb{Z}/p\mathbb{Z}$-quotient singularity.

Known Properties of Intersection Matrices

Attached to the resolution π is its dual graph Γ_N, with vertices $v_1, \ldots, v_n$, where v_i and v_j are linked by $(C_i \cdot C_j)_X$ distinct edges when $i \neq j$.

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Let $\mathrm{Ad}(\Gamma_N)$ denote the adjacency matrix of the graph Γ_N. The matrix N has the form $N = \mathrm{Diag}(c_{11}, \dots, c_{nn}) + \mathrm{Ad}(\Gamma_N)$, where $c_{ii} = (C_i \cdot C_i)_X$ is the self-intersection number of C_i.

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Theorem (L.): When the exceptional divisor of π has smooth components with normal crossings, the components C_i/k are smooth projective lines and the graph Γ_N is a tree.

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Theorem (L.): *The discriminant group $\Phi_N := \mathbb{Z}^n/\mathrm{Im}(N)$ is an elementary abelian p-group. In particular $|\Phi_N| = |det(N)| = p^s$ for some integer $s \geq 0$.*

Known Properties of Intersection Matrices

Let Γ be a finite connected tree. A symmetric matrix N of the form $N = \mathrm{Ad}(\Gamma) - \mathrm{Diag}(c_1, \ldots, c_n)$ with $c_1, \ldots, c_n \geq 1$ is not necessarily negative definite.

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Wagreich (1970): $|Z \cdot Z| \leq$ multiplicity of $(B, \mathfrak{m}_B)$.

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• and such that up to a choice of the ordering of the irreducible components C_i, the intersection matrix associated with E is equal to the given matrix N.

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Problem. Characterize the p-suitable intersection matrices which arise from a $\mathbb{Z}/p\mathbb{Z}$-quotient singularity.

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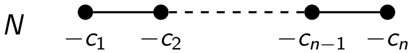

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For each p, there are *finitely many* such matrices, and *every such matrix N arises* as a cyclic quotient singularity.

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Addendum. The dual graphs of the resolutions of the singularities occurring in the above theorem are all star-shaped trees.

Graphs with many nodes

Open Question.
For each prime p, and for every integer $\delta \geq 2$, exhibit a quotient singularity $\mathrm{Spec}\, A^{\mathbb{Z}/p\mathbb{Z}}$ whose associated dual graph Γ_N has δ nodes.

Example with five nodes (personal record).

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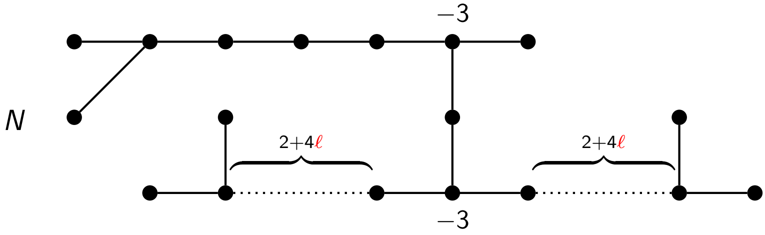
Magma: ...whose resolution has $18 + 8\ell$ components



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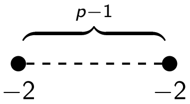

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Problem. Is there a lower bound $n_0(p)$, with $\limsup n_0(p) = \infty$, such that if N is a p-suitable matrix of size n which occurs as a $\mathbb{Z}/p\mathbb{Z}$-quotient singularity and whose associated tree has at least one node, then $n \geq n_0(p)$.

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Theorem (Artin, 1977) *The Dynkin diagram E_8, with $\Phi_{E_8} = (1)$ and $n = 8$, occurs as a $\mathbb{Z}/p\mathbb{Z}$-quotient singularity only for $p = 2$ and 5.*

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Theorem (L.) *Fix any prime p and any integer $\delta \geq 2$. Then there exists a p-suitable intersection matrix N whose associated graph has δ nodes and $|\Phi_N| \geq p^{\delta}$.*

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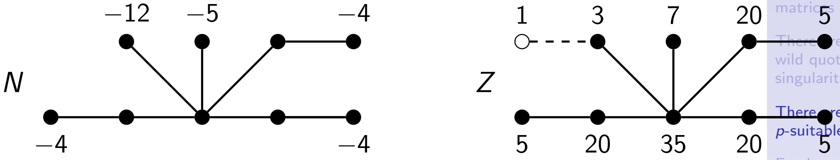

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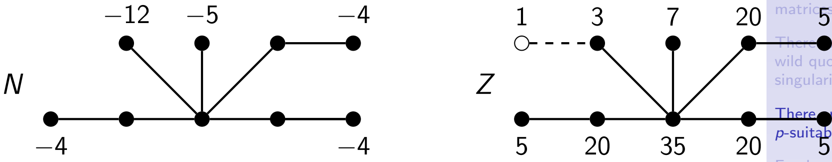

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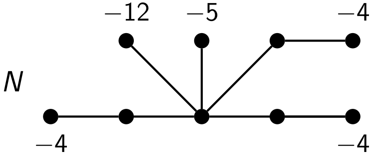

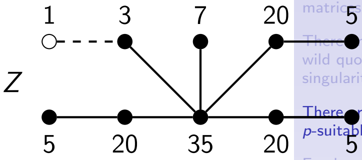

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Theorem (L.-Schröer). $z^p + x^{pr+1} + y^{ps+1} = 0$ is a $\mathbb{Z}/p\mathbb{Z}$-quotient singularity.

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Theorem (L.) *Assume that $p = 2$. Then either $-Z$ or $-Z/2$ is a column of N^{-1}.*

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Motivation for considering this question is the following:

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The set-up of models of curves is interesting as it produces examples of quotient singularities in both the equicharacteristic case, and the mixed characteristic case. It also produces examples of singularities that are not hypersurface singularities.

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The map ρ is Galois of order $|H|$, and the second map is the normalization map of $\mathcal{Z}_k^{red}$.

Let $P_1, \ldots, P_d$, be the ramification points of the map $\mathcal{Y}_k \to \mathcal{Y}_k/\langle\sigma\rangle$. We assume $d \geq 1$.

Let $Q_1, \ldots, Q_d$ be their images in $\mathcal{Z}$.

The normal scheme $\mathcal{Z}$ is singular exactly at $Q_1, \ldots, Q_d$.

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Recap: Does Property (b) hold for any $\mathbb{Z}/p\mathbb{Z}$-quotient singularity when the graph has at least one node?

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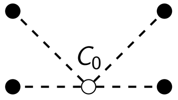

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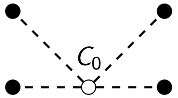

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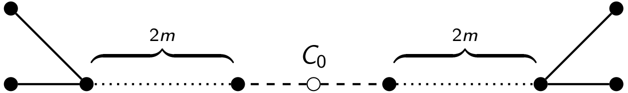

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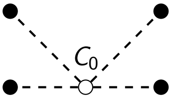

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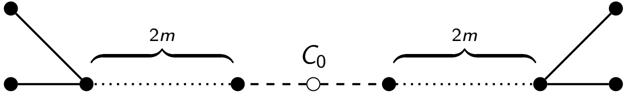

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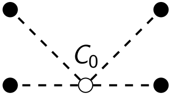

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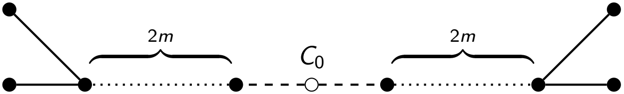

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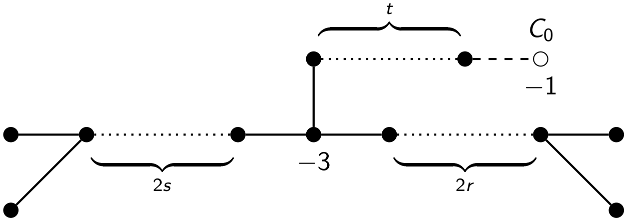

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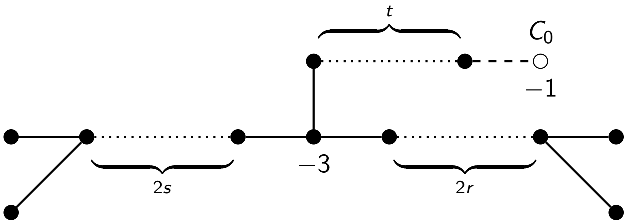

We must have $r + s$ even. Maybe $r = s$ always? $t = 0$?

THANKS!

Merci!

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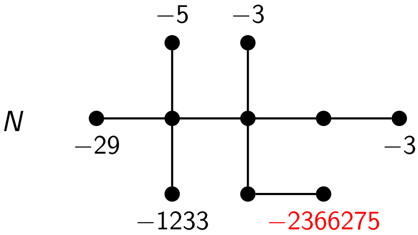

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